

## PHYS 320 Analytical Mechanics

Fall 2018

### Homework Assignment # 18

☞ Read Taylor Sections 6.1 - 6.4, 7.1

⊛ Taylor Problems: 5.52

⊛ Extra Questions:

- I. Suppose a step function force,  $F_{ext}(t) = F_o H(t)$ , is applied to a damped oscillator at a time  $t_o$  as described by the Heaviside function:

$$H(t, t_o) = \begin{cases} 0, & t < t_o \\ a, & t > t_o \end{cases}$$

The solution to the step force function applied to a damped SHO can be written as

$$x(t) = \frac{H(t, t_o)}{a} \left\{ e^{-\gamma(t-t_o)} [A_1 \cos(\omega_d(t-t_o)) + A_2 \sin(\omega_d(t-t_o))] + \frac{a}{\omega_o^2} \right\}$$

A. Verify by direct substitution that this is a solution to the damped driven oscillator equation for this  $F_{ext}$ .

B. Assuming  $x(t < t_o) = 0$  and with initial conditions  $x(t_o) = 0$  and  $dx(t_o)/dt = 0$ , find  $A_1$  and  $A_2$ . Plot the response,  $x(t)$ , of the oscillator using  $\gamma = 0.2/s$ ,  $\omega_o = 2 \text{ rad/s}$ ,  $a = 2$ ,  $F_o = 1N$ , and  $t_o = 2s$  from  $t = 0$  to  $t = 30s$  using Maple.

C. Next, consider the impulse function:

$$I(t_o, t_1) = H(t_o) - H(t_1) = \begin{cases} 0, & t < t_o \\ a, & t_o < t < t_1 \\ 0, & t > t_1 \end{cases}$$

where  $\tau = t_1 - t_o$ . Find the solution for  $t > t_1$ .

Finally, plot the total response for  $t_o = 0$ ,  $\gamma = 0.2 \omega_o$ , and  $\tau = 5(2\pi/\omega_d)$  using Maple.